

Leptonic constant from B meson radiative decay

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Abstract

We propose a method to determine the leptonic decay constant f_{B^*} in the infinite heavy quark mass limit from the analysis of the radiative decay mode $B^- \rightarrow \ell^- \bar{\nu}_\ell \gamma$. The method relies on HQET symmetries and on experimental data from $D^{*0} \rightarrow D^0 \gamma$.

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The measurement of f_B and f_{B^*} , the leptonic decay constants of the B and B^* mesons, defined by the matrix elements:

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 q | B(p) \rangle = i p_\mu f_B \quad (1)$$

$$\langle 0 | \bar{b} \gamma_\mu q | B^*(p, \epsilon) \rangle = m_{B^*} f_{B^*} \epsilon_\mu , \quad (2)$$

represents one of the main goals of the current and future experimental investigations in the heavy quark physics. The reason can be found in the prime role played by f_B in the hadronic systems containing one heavy quark. To give an example, f_B^2 appears in the formula relating the mass difference between the B^0 -meson mass eigenstates to $|V_{td}|^2$ in the box diagram computation of the $B^0 - \bar{B}^0$ mixing; therefore, the size of the unitarity triangle and the analysis of possible CP violations in the B systems crucially depend on the value of this hadronic parameter.

As a second example of the relevance of the leptonic B constant, we can consider the Heavy Quark Effective Theory applied to the physical world of the heavy hadrons. In the limit $m_b \rightarrow \infty$, f_B and f_{B^*} scale according to the relation

$$f_B = f_{B^*} = \frac{\hat{F}}{\sqrt{m_B}} . \quad (3)$$

The parameter \hat{F} , independent (modulo logarithmic corrections) of the heavy quark mass, represents a low-energy parameter related to the non-perturbative dynamics of light quark and gluon degrees of freedom, and plays a role analogous to f_π in chiral theories for light hadrons.

On the theoretical side, much effort has been devoted to the calculation of $f_B = \frac{\hat{F}}{\sqrt{m_B}}$ by non-perturbative methods such as lattice QCD [1] and QCD sum rules [2]; for example, a QCD sum rules analysis in the infinite heavy quark mass limit provides us with the value:

$$\hat{F} = 0.25 - 0.45 \text{ GeV}^{\frac{3}{2}} . \quad (4)$$

(depending on the inclusion of α_s corrections).

On the experimental side, the most natural process to measure f_B would be the purely leptonic decay mode $B^- \rightarrow \ell^- \bar{\nu}_\ell$, whose decay width is given by:

$$\Gamma(B^- \rightarrow \ell^- \bar{\nu}_\ell) = \frac{G_F^2}{8\pi} |V_{ub}|^2 f_B^2 \left(\frac{m_\ell}{m_B} \right)^2 m_B^3 \left(1 - \frac{m_\ell^2}{m_B^2} \right)^2 . \quad (5)$$

If one determines $|V_{ub}|$ from other processes, for example from the end-point spectrum of the charged lepton in the B meson inclusive semileptonic decay [3, 4], one can obtain f_B by this equation. The difficulty is represented by the helicity suppression displayed in Eq. (5). Since the lepton pair must be a spin 0 state and the antineutrino has a right-handed helicity, also the charged lepton is forced to be right-handed. The effect is the suppression factor $(\frac{m_\ell}{m_B})^2$ that makes the purely leptonic decay mode hardly accessible in the electron and in the muon case. As a matter of fact, using the value for the B^- lifetime $\tau_{B^-} = 1.646 \pm 0.063 \text{ ps}$ [4], the expected rates are:

$$\mathcal{B}(B^- \rightarrow e^- \bar{\nu}_e) \simeq 6.6 \left[\frac{V_{ub}}{0.003} \right]^2 \left[\frac{f_B}{200 \text{ MeV}} \right]^2 10^{-12} \quad (6)$$

$$\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) \simeq 2.8 \left[\frac{V_{ub}}{0.003} \right]^2 \left[\frac{f_B}{200 \text{ MeV}} \right]^2 10^{-7} , \quad (7)$$

to be compared with the experimental upper bound put by CLEO [5]:

$$\mathcal{B}(B^- \rightarrow e^- \bar{\nu}_e) < 1.5 \cdot 10^{-5} \quad (8)$$

$$\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) < 2.1 \cdot 10^{-5} \quad (9)$$

(at 90 % confidence level).

As for the channel $B \rightarrow \tau \nu_\tau$, the helicity suppression is absent and the expected rate is larger:

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) \simeq 6.8 \left[\frac{V_{ub}}{0.003} \right]^2 \left[\frac{f_B}{200 \text{ MeV}} \right]^2 10^{-5} . \quad (10)$$

However, the τ identification puts a hard experimental challenge. The upper limits found by CLEO [5] and ALEPH [6] Collaborations read:

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) < 2.3 \cdot 10^{-3} \quad (11)$$

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) < 1.8 \cdot 10^{-3} \quad (12)$$

(at 90 % confidence level), respectively.

For these reasons it is worthwhile to search for other paths, and analyze other possible decay modes that are sensitive to the value of \hat{F} . For example, one could use flavour symmetry and consider a measurement of f_{B^*} from the spectrum of the semileptonic decay $B \rightarrow \pi \ell \nu_\ell$ near zero recoil [7, 8] compared to the spectrum of $D \rightarrow \pi \ell \nu_\ell$ in the same kinematics. In this case, however, one has to face a strong phase-space suppression.

Another possibility, suggested in refs.[9] and [10], is provided by the radiative leptonic decay channel

$$B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma \quad (13)$$

which does not suffer of the helicity suppression because of the presence of the photon in the final state. In this case there are several uncertainties related to the hadronic parameters appearing in the matrix element governing the decay mode (13). Within such uncertainties, the branching ratio of the decay (13) has been estimated in the range $10^{-7} - 10^{-6}$ (in the case of light leptons), thus making the channel promising for a future B -factory. We shall now study how this decay mode can be used to measure f_{B^*} .

In order to analyze the dependence of the amplitude for the process (13) on f_{B^*} , let us consider the diagrams which describe it; they can be divided into two classes. The first class consists of structure dependent (SD) diagrams such as those of Figs. 1; the second class contains bremsstrahlung diagrams where the photon is emitted from the B^- or from the charged lepton leg.

The bremsstrahlung amplitude is given by:

$$\mathcal{M}_B = if_{B^*} \frac{G_F}{\sqrt{2}} V_{ub} m_\mu [(F(k^2) \frac{\epsilon \cdot p}{p \cdot k} - \frac{\epsilon \cdot p_\ell}{p_\ell \cdot k}) \bar{\mu} (1 - \gamma_5) \nu - \frac{1}{2p_\ell \cdot k} \bar{\mu} \not{\epsilon} \not{k} (1 - \gamma_5) \nu] \quad (14)$$

where p, p_ℓ, k are the momenta of B^- , μ and γ , respectively, ϵ is the photon polarization vector, and $F(k^2)$ is the B^- electromagnetic form factor. This contribution vanishes in the limit $m_\mu \rightarrow 0$ and we shall make this approximation, so that the relevant diagrams governing (13) are the SD diagrams. We shall suppose, as in [10], that in these polar diagrams the intermediate state can be a $J^P = 1^-(B^*)$ and a positive parity B^{**} meson. The amplitude with intermediate $P(= B^*, B^{**})$ state can be written as follows:

$$\mathcal{M}_{SD}^{(P)} = \frac{G_F}{\sqrt{2}} V_{ub} \mathcal{A}(B \rightarrow P\gamma) \frac{i}{(p-k)^2 - m_P^2} < 0 | \bar{u} \gamma^\mu (1 - \gamma_5) b | P > l_\mu, \quad (15)$$

where $l_\mu = \bar{\ell}(p_\ell) \gamma_\mu (1 - \gamma_5) \nu(p_\nu)$ is the lepton current, $\mathcal{A}(B \rightarrow P\gamma)$ is the amplitude of the process $B \rightarrow P\gamma$, and P indicates the pole. From Eq.(15) (with $P = B^*$) it is clear that, for light leptons in the final state, the radiative B decay can give access to the decay constant f_{B^*} provided that

- 1) one has enough information on the amplitude $\mathcal{A}(B^* \rightarrow B\gamma)$,
- 2) the remaining part of the SD contribution $\mathcal{M}_{SD}^{(B^{**})}$ is small compared to $\mathcal{M}_{SD}^{(B^*)}$.

Let us begin by discussing the first point and let us consider the contribution of the B^* pole to (15). In computing the on-shell amplitude $\mathcal{A}(B^* \rightarrow B\gamma)$ one has to take into account the coupling of the electromagnetic current to the heavy quark and to the light quark, i.e. the terms arising from the decomposition $J_\mu^{em} = e_b \bar{b} \gamma_\mu b + e_q \bar{q} \gamma_\mu q$ (e_b, e_q = quark charges).

The b -quark contribution is described (in the limit $m_b \rightarrow \infty$) by the amplitude:

$$\epsilon^{*\mu} \langle B(v') | e_b \bar{b} \gamma_\mu b | B^*(v, \eta) \rangle = i e_b \xi(v \cdot v') \sqrt{m_B m_{B^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu} \eta^\nu v^\alpha v'^\beta, \quad (16)$$

where v and v' are B^* and B four-velocities, respectively, and $\xi(v \cdot v')$ is the universal Isgur-Wise form factor (IW) [11]. For on-shell B and B^* , since $v \cdot v' = \frac{m_B^2 + m_{B^*}^2}{2m_B m_{B^*}} \simeq 1$, we can use the normalization of the IW function: $\xi(1) = 1$.

The second contribution: $\epsilon^{*\mu} \langle B | e_q \bar{q} \gamma_\mu q | B^* \rangle$ represents the coupling of the electromagnetic current to the light quark q ($q = u$ for B^- decay); this contribution dominates in the $m_b \rightarrow \infty$ limit, and is more uncertain since it cannot be estimated within HQET because it involves light degrees of freedom. On the experimental side we have no data on the width $B^* \rightarrow B\gamma$ at the moment, and it is unlikely it will be measured in the near future. On the other hand, the experimental $D^{*0,-} \rightarrow D^{0,-}\gamma$ branching ratios are known (even though the full $D^{*0,-}$ width has not been measured yet) and we may presume that in future we will get information on the D^* partial radiative width.

Our proposal is to use these pieces of information to obtain $B^* \rightarrow B\gamma$.

The D^{*0} radiative width is given by:

$$\Gamma(D^{*0} \rightarrow D^0 \gamma) = \frac{q_\gamma^3}{12\pi} \frac{m_{D^{*0}}}{m_{D^0}} g_{D^* D \gamma}^2 \quad (17)$$

where q_γ is the photon momentum in the D^* rest frame and

$$g_{D^* D \gamma} = e \left[\frac{e_c}{m_{D^*}} + \frac{e_q}{\Lambda_q} \right] \quad (18)$$

($e_c = 2/3$, $e_q = e_u = 2/3$). A measurement of $\Gamma(D^{*0} \rightarrow D^0 \gamma)$ would provide a determination of the mass constant Λ_q that parametrizes the matrix element $\langle D | \bar{q} \gamma_\mu q | D^* \rangle$. On the other hand

$$\mathcal{A}(B^*(v, \eta) \rightarrow B(v') \gamma(q, \epsilon)) = i e \left[\frac{e_b}{m_{B^*}} + \frac{e_q}{\Lambda_q} \right] \times m_{B^*} \sqrt{m_B m_{B^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu} \eta^\nu v^\alpha v'^\beta, \quad (19)$$

and therefore, by the knowledge of Λ_q , one would obtain the matrix element needed to compute Eq.(15) (with $P = B^*$). In other words with the approach we have just described, we might extract \hat{F} by the measurement of *both* $\Gamma(D^{*0} \rightarrow D^* \gamma)$ and $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$; we shall discuss below the sensitivity of this approach.

In order to assess the reliability of this method we shall also discuss as a consistency test based on information in part already available, i.e. data on $B \rightarrow \pi \ell \nu$ [12], $D \rightarrow K \ell \nu$ [13] and on the BR 's $\mathcal{B}(D^* \rightarrow D\pi)$ and $\mathcal{B}(D^* \rightarrow D\gamma)$ [13]. Let us consider the partial width

$$\Gamma(D^{*0} \rightarrow D^0 \pi^0) = \left(\frac{g_{D^* D \pi}}{\sqrt{2}} \right)^2 \frac{q_\pi^3}{24\pi m_{D^*}^2} \quad (20)$$

where q_π is the pion momentum in the D^* rest frame and $g_{D^* D \pi}$ is the strong $D^* D \pi$ coupling constant, which, in the heavy quark mass limit, is given by

$$g_{D^* D \pi} = g_{D^{*+} D^0 \pi^+} = \frac{2g}{f_\pi} \sqrt{m_D m_{D^*}}. \quad (21)$$

The ratio of the partial widths (20) and (17) is experimentally known [13]

$$R = \frac{\mathcal{B}(D^{*0} \rightarrow D^0 \pi^0)}{\mathcal{B}(D^{*0} \rightarrow D^0 \gamma)} = 1.75 \pm 0.21, \quad (22)$$

from which the ratio of the D^* decay constants can be obtained:

$$\frac{g}{g_{D^* D \gamma}} = 0.808 \sqrt{R} \text{ GeV}. \quad (23)$$

In order to extract $g_{D^* D \gamma}$ from Eq.(23) one needs the strong coupling constant g ; several determinations of g have appeared in the literature, based on QCD sum rules [8, 14, 15] or potential models [16]; they indicate a value in the range 0.2 – 0.4. We shall avoid, however, to rely on these estimates and we shall try to minimize the theoretical bias using experimental data of the decay $D \rightarrow K \ell \nu$ ($\mathcal{B}(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = 6.6 \pm 0.9 \cdot 10^{-2}$ [13]) and the recently observed decays [12]

$$\mathcal{B}(B \rightarrow \pi \ell \nu) = 1.70 \pm 0.50 \cdot 10^{-4} \quad (BSW) \quad (24)$$

$$\mathcal{B}(B \rightarrow \pi \ell \nu) = 1.19 \pm 0.65 \cdot 10^{-4} \quad (ISGW). \quad (25)$$

The two determinations refer to the model used in the Montecarlo code to compute the efficiencies: the Bauer et al. model [17] or the Isgur et al. model [18].

For these decays, assuming a simple pole formula for the form factors $f_+^{B \rightarrow \pi}(q^2)$ and $f_+^{D \rightarrow K}(q^2)$, which is what is experimentally found for $D^+ \rightarrow \bar{K}^0 \ell^+ \nu_\ell$ and generally accepted on theoretical grounds [19, 20, 21, 22] for $B \rightarrow \pi \ell \nu$, one gets the following results. The semileptonic $B \rightarrow \pi \ell \nu$ partial width is given by

$$\Gamma(B \rightarrow \pi \ell \nu) = \hat{F}^2 g^2 |V_{ub}|^2 \frac{G_F^2}{192\pi^3 f_\pi^2 m_B^4} J \quad (26)$$

with

$$J = \int_0^{q_{max}^2} dq^2 \frac{\lambda^{3/2}(m_B^2, m_\pi^2, q^2)}{(1 - q^2/m_{B^*}^2)^2}. \quad (27)$$

In (27) λ is the triangular function. For $D \rightarrow K\ell\nu_\ell$ one has to change $m_B \rightarrow m_D$, $V_{ub} \rightarrow V_{cs}$, $m_{B^*} \rightarrow m_{D^*}$, $m_\pi \rightarrow m_K$. From Eq.(27) one gets:

$$\hat{F}g = \frac{2.95 \cdot 10^{-2}}{|V_{ub}|} \left[\mathcal{B}(\bar{B}^0 \rightarrow \pi^- \ell \nu) \right]^{1/2} GeV^{\frac{3}{2}} = (1.07 - 1.28) \cdot 10^{-1} \frac{1}{\left| \frac{V_{ub}}{0.003} \right|} GeV^{3/2}, \quad (28)$$

$$\hat{F}g = \frac{0.45}{|V_{cs}|} \left[\mathcal{B}(\bar{D}^+ \rightarrow K^0 e^+ \nu_e) \right]^{1/2} GeV^{\frac{3}{2}} = 1.18 \cdot 10^{-1} GeV^{3/2}; \quad (29)$$

thus, the use of $B \rightarrow \pi\ell\nu_\ell$ data or $D \rightarrow K\ell\nu_\ell$, employing light and heavy flavour symmetries, give similar results. Eqs. (28,29) allow to obtain, from (23), $g_{D^*D\gamma}$ and therefore, from (18), Λ_q as a function of \hat{F} .

We shall use below this approach as a consistency test of our method. In both cases, the proposed method and the consistency test, after having put the value of Λ_q into (19), we can compute $\mathcal{A}(B^* \rightarrow B\gamma)$ and the amplitude $\mathcal{M}_{SD}^{(B^*)}$, i.e. the contribution of the B^* pole to the decay $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$. The final expression for the amplitude $\mathcal{M}_{SD}^{(B^*)}$ is as follows:

$$\mathcal{M}_{SD}^{(B^*)} = \frac{C_1 f_{B^*}}{(v \cdot k + \Delta)} \epsilon_{\mu\sigma\alpha\beta} l^\mu \epsilon^{*\sigma} v^\alpha k^\beta, \quad (30)$$

where $\Delta = m_{B^*} - m_B$, and C_1 is given by:

$$C_1 = \frac{G_F}{\sqrt{2}} V_{ub} \frac{m_{B^*}}{2m_B} \sqrt{m_B m_{B^*}} e \left[\frac{e_b}{m_{B^*}} + \frac{e_q}{\Lambda_q} \right]. \quad (31)$$

From (30) we can compute the contribution of the B^* pole to $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ as a function of the parameter \hat{F} .

Let us study the effect of the B^{**} pole. It is well known that in the limit of infinitely massive heavy quarks ($m_Q \rightarrow \infty$) the strong dynamics of the heavy quark decouples from the dynamics of light degrees of freedom, with the consequence that the spin of the heavy quark and the spin of light degrees of freedom are separately conserved. In the case of the first orbitally excited heavy states, with orbital angular momentum $\ell = 1$, the total angular momentum of the light degrees of freedom can be $s_\ell = 1/2$ or $s_\ell = 3/2$. To each of these values corresponds a doublet of positive parity heavy mesons: the doublet (B_0, B'_1) with $J^P = (0^+, 1^+)$ in correspondence to $s_\ell = 1/2$, and the doublet (B_1, B_2) with $J^P = (1^+, 2^+)$ ($s_\ell = 3/2$).

In the charm sector only the members of the $s_\ell = 3/2$ doublet (both in the strange and non-strange charmed channels) have been observed: in the non-strange case they are the states $D_1(2420)$ and $D_2(2460)$ decaying to $D\pi$, $D^*\pi$ with pions in D -wave, therefore with small decay widths ($\Gamma_{D_1} = 18_{-4}^{+6} \text{ MeV}$ and $\Gamma_{D_2} = 23 \pm 10 \text{ MeV}$ [13]). The states $s_\ell = 1/2$ can decay into $D\pi$, $D\pi^*$ with pions in S -wave, and their large decay width makes their observation rather problematic. In the beauty sector, recent results from LEP Collaborations show the existence of positive parity orbitally excited B mesons (B^{**}) with an average mass $m_{B^{**}} = 5732 \pm 5 \pm 20 \text{ MeV}$ [23].

Only the axial vector states 1^+ can contribute as poles to the decay (Fig. 1 b); moreover, the state B_1 , having $s_\ell = 3/2$, has vanishing coupling to the weak current in the limit $m_b \rightarrow \infty$ [22], and only the state B'_1 (with $s_\ell = 1/2$) gives a contribution in the same limit.

As to B'_1 , following the same steps leading to Eq.(30) we get:

$$\mathcal{M}_{SD}^{(B'_1)} = i \frac{C_2 f_{B'_1}}{(v \cdot k + \Delta')} (\epsilon \cdot v k_\mu - v \cdot k \epsilon_\mu) l^\mu \quad (32)$$

where $\Delta' = m_{B'_1} - m_B$,

$$< 0 | \bar{b} \gamma_\mu \gamma_5 q | B'_1(p, \eta) > = f_{B'_1} m_{B'_1} \eta_\mu, \quad (33)$$

and

$$C_2 = \frac{G_F}{\sqrt{2}} V_{ub} \frac{m_{B'_1}}{2m_B} \sqrt{m_B m_{B'_1}} e \left[\frac{2e_b \tau_{1/2}(1)}{m_B} + \frac{e_q}{\Lambda'_q} \right]. \quad (34)$$

The function $\tau_{1/2}(v \cdot v')$ is the universal form factor, analogous to the IW function, describing the matrix element of the weak current between positive parity heavy mesons and the doublet (B, B^*) in the limit $m_b \rightarrow \infty$ [24]; it can be defined as follows:

$$< B'_1(v', \eta) | \bar{b} \gamma_\mu \gamma_5 b | B(v) > = i \tau_{1/2}(v \cdot v') \sqrt{m_B m_{B'_1}} \epsilon_{\mu\nu\alpha\beta} \eta^{*\nu} v^\alpha v'^\beta, \quad (35)$$

and its value at $v \cdot v' = 1$ has been estimated by QCD sum rules [25]: $\tau_{1/2}(1) \simeq 0.24$. We notice that, because of the factor $1/m_B$ and the small value of $\tau_{1/2}(1)$, the first term in the r.h.s. of (35) is expected to be small as compared to the second one.

Putting together the contributions of B^* and B'_1 , we get:

$$\begin{aligned} \Gamma(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) &= \frac{2}{3(2\pi)^3} \int_0^{m_B/2} dE_\gamma E_\gamma^3 (m_B - 2E_\gamma) \left[\frac{|C_1|^2 f_{B^*}^2}{(E_\gamma + \Delta)^2} + \frac{|C_2|^2 f_{B'_1}^2}{(E_\gamma + \Delta')^2} \right] \\ &= \Gamma^{(B^*)} + \Gamma^{(B'_1)}. \end{aligned} \quad (36)$$

The relative contribution of the B^* and of the B'_1 poles is given by:

$$\frac{\Gamma(B'_1)}{\Gamma(B^*)} = \frac{PS^{(B'_1)}}{PS^{(B^*)}} \times \frac{|C_2 f_{B'_1}|^2}{|C_1 f_{B^*}|^2} \quad (37)$$

therefore it is weighted by the ratio of the phase-space coefficients

$$\frac{PS^{(B'_1)}}{PS^{(B^*)}} \simeq 0.54 \quad (38)$$

(using the experimental values of the mass differences $\Delta = 46 \text{ MeV}$ and $\Delta' \simeq 500 \text{ MeV}$) and depends on the ratio $(C_2 f_{B'_1})/(C_1 f_{B^*})$. This last ratio is basically determined by the coupling of the electromagnetic current to the light quarks. This can be seen from Eqs. (31,34) neglecting $1/m_b$ terms, since the heavy quark magnetic momentum is subleading in the inverse heavy quark mass expansion:

$$\frac{C_2}{C_1} \frac{f_{B'_1}}{f_{B^*}} \simeq \left(\frac{m_{B'_1}}{m_{B^*}}\right)^{\frac{3}{2}} \frac{f_{B'_1}}{f_{B^*}} \frac{\Lambda_q}{\Lambda'_q}. \quad (39)$$

The ratio $\frac{f_{B'_1}}{f_{B^*}} \frac{\Lambda_q}{\Lambda'_q}$ can be estimated using experimental results from semileptonic D -meson decay channels [22, 26]. As a matter of fact, one can make use of the Vector Meson Dominance (VMD) [26] ansatz and assume that the coupling of the electromagnetic current to the light quarks is mediated by vector meson states V ($V = \rho, \omega$). By this assumption the matrix elements $\langle B|\bar{q}\gamma^\mu q|B^* \rangle$ and $\langle B|\bar{q}\gamma^\mu q|B'_1 \rangle$ can be written as:

$$\begin{aligned} \langle B(p')|\bar{q}\gamma_\mu q|P(p, \epsilon_2) \rangle &= \sum_{V, \eta} \langle B(p')V(q, \epsilon_1(\eta))|P(p, \epsilon_2) \rangle \frac{i}{q^2 - m_V^2} \\ &\quad \langle 0|\bar{q}\gamma_\mu q|V(q, \epsilon_1(\eta)) \rangle, \end{aligned} \quad (40)$$

where $P = B^*, B'_1$ and therefore they are proportional to the strong couplings $\langle B|V|B^* \rangle$ and $\langle B|V|B'_1 \rangle$; these matrix elements have been estimated in [22] in the framework of an effective chiral theory for heavy mesons, using experimental information from the $D \rightarrow K^* \ell \nu$ semileptonic decay and the (approximate) symmetries of the effective theory (heavy flavour symmetry and flavour $SU(3)$ symmetry for the light degrees of freedom):

$$\langle B(v')V(k, \epsilon)|B^*(v, \eta) \rangle = \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu^* \eta_\nu v_\alpha v'_\beta \sqrt{m_B m_{B^*}} m_{B^*} 2\sqrt{2} g_V \lambda \quad (41)$$

$$\langle B(v')V(k, \epsilon)|B'_1(v, \eta) \rangle = [(k \cdot v)(\epsilon^* \cdot \eta) - (k \cdot \eta)(\epsilon^* \cdot v)] \sqrt{m_B m_{B'_1}} 2\sqrt{2} g_V \mu \quad (42)$$

where $\lambda = -0.4 \text{ GeV}^{-1}$, $\mu = -0.13 \text{ GeV}^{-1}$ and $g_V = 5.8$ [22]. As a result one finds

$$\frac{f_{B'_1}}{f_{B^*}} \frac{\Lambda_q}{\Lambda'_q} \simeq 0.4 \quad . \quad (43)$$

Therefore, we conclude that

$$\frac{\Gamma(B'_1)}{\Gamma(B^*)} \simeq 0.1 \quad (44)$$

i.e. the B'_1 pole represents a contribution at the level of 10% of the radiative $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ width. The phase-space suppression is even larger for the contribution of other (radially or orbitally) excited B states, which therefore can be safely neglected. Then, we can assume that the diagram of Fig. 1b adds a term of the order of 10% as compared to Fig. 1a; therefore we shall take into account it by multiplying the contribution of the B^* pole to the width by a normalization factor $K = 1.1$. We are aware that this estimate of the diagrams of Fig. 1b is more uncertain than the evaluation of Fig. 1a. Nevertheless we observe that the VMD hypothesis successfully describes a number of low energy phenomena involving photon radiation [27]; moreover these theoretical uncertainties will not affect strongly the final determination of $\mathcal{B}(B \rightarrow \mu \nu \gamma)$, since, in any case, the contribution of Fig. 1b is much smaller than the B^* term.

The main difference of the above analysis with respect to ref. [10] consists in the evaluation of the coupling $g_{BB'_1\gamma}$ and in the role of $g_{B^*B\gamma}$. In [10] the non relativistic quark model has been employed to compute $g_{B^*B\gamma}$, using $\Lambda_q = 330 \text{ MeV} \simeq m_u$ (constituent mass of the light quarks); moreover, it has also been adopted to estimate $g_{B'_1B\gamma} = \frac{g_{B^*B\gamma}}{\sqrt{3}}$. On the other hand, in our approach $g_{B^*B\gamma}$ is a quantity that should be inferred from experimental data, thus reducing the dependence of the results on the hadronization model. The prediction in [10] for $\mathcal{B}(B \rightarrow \mu \nu \gamma)$ is in the range $10^{-7} - 10^{-6}$.

Let us now come to the numerical results. As we observed above, to set definite predictions one needs the experimental input $\Gamma(D^{*0} \rightarrow D^0 \gamma)$, which is not available yet. In order to test the sensitivity of the method, we have used two theoretical determinations of the radiative D^* width: $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 22 \text{ KeV}$ (ref.[16]) and $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 11 \text{ KeV}$ (ref.[28]); with these input data, which represent rather extreme cases (an intermediate prediction can be found in [26]), and using $V_{ub} = 0.003$, we get the curves (a) and (b) depicted in Fig.2. From this figure one can see that the branching ratio of the radiative decay mode $B^- \rightarrow \mu^- \nu \gamma$ is expected to be larger than in the purely leptonic mode. This can be appreciated by comparing the prediction in Eq.(7) with the outcome in Fig.2 in correspondence to the value $\hat{F} \simeq 0.45 \text{ GeV}^{3/2}$: $\mathcal{B}(B^- \rightarrow \mu^- \nu \gamma) = 4 - 10 \times 10^{-7}$ (depending on the $D^{*0} \rightarrow D^0 \gamma$ decay width). This implies that the radiative mode, for quite large values of the leptonic constant \hat{F} is favoured by a factor 2–3 (at least) with respect to the purely leptonic mode $B^- \rightarrow \mu^- \nu$; moreover, as far as the statistics is concerned,

the use of radiative decay allows to employ the electron channel as well, which provides a gain of an additional factor of two.

To test this result using the semileptonic data (Eqs. 28, 29), we have reported in Fig.3 two curves corresponding to the central values for $B \rightarrow \pi \ell \nu$ given in Eq.(24) and (25); for the range of \hat{F} in Eq.(4) we obtain $\mathcal{B}(B^- \rightarrow \mu^- \nu \gamma) \simeq 2 - 3 \times 10^{-7}$. The data for $D \rightarrow K \ell \nu$ give an intermediate result. As we have stressed above, the outcome in Fig.3 is based not only on experimental data, but also on additional theoretical assumptions, such as the polar dependence of the semileptonic $B \rightarrow \pi$ form factor. So, such results are characterized by a quite large uncertainty; nevertheless, for intermediate values of \hat{F} there is an overlap region with the result in Fig.2 that allows us to conclude that the consistency test does not contradict our main results in Fig.2

In conclusion, our analysis confirms that the decay channel $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ can be used as a way to measure the leptonic B^* decay constant; one expects larger decay rates than in the purely leptonic case, even though the detection of the photon in the final state may reduce the reconstruction efficiency. Moreover, since the method described here is strongly based on HQET symmetries, it would be useful in any case to look for the radiative leptonic B decay to test experimentally HQET predictions in this context.

Note added

After completing this work we became aware of the paper [29] where the calculation of $B \rightarrow \ell \nu \gamma$ is performed by light-cone sum rules. The result confirms the expectation that the radiative decay rates are larger than the purely leptonic rates.

Acknowledgments

We thank N.Paver for useful discussions.

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Figure Captions

Figure 1

Diagrams dominating the $B^- \rightarrow \ell^- \bar{\nu}_\ell \gamma$ decay mode in the limit $m_\ell \rightarrow 0$. B^* is the vector meson state, B^{**} is the 1^+ axial vector meson state.

Figure 2

Branching ratio $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ obtained according to the method described in the text. The curves (a) and (b) refer to the values: $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 22 \text{ KeV}$ [16] (continuous line) and $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 11 \text{ KeV}$ [28] (dashed line).

Figure 3

Branching ratio $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ obtained using data on the semileptonic $B \rightarrow \pi \ell \nu$ decay. The curves (a) and (b) refer to the input values: $\mathcal{B}(B \rightarrow \pi \ell \nu) = 1.70 \cdot 10^{-4}$ (continuous line) and $\mathcal{B}(B \rightarrow \pi \ell \nu) = 1.19 \cdot 10^{-4}$ (dashed line).

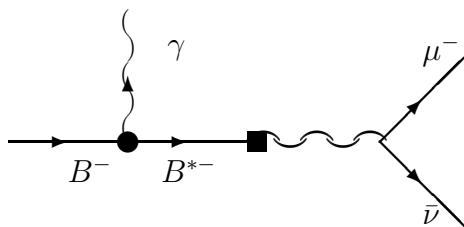


Fig. 1 a)

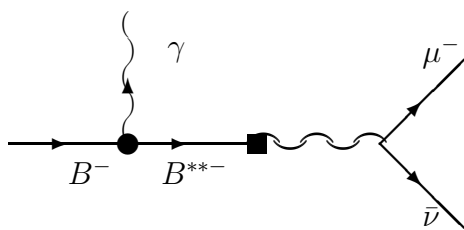


Fig. 1 b)

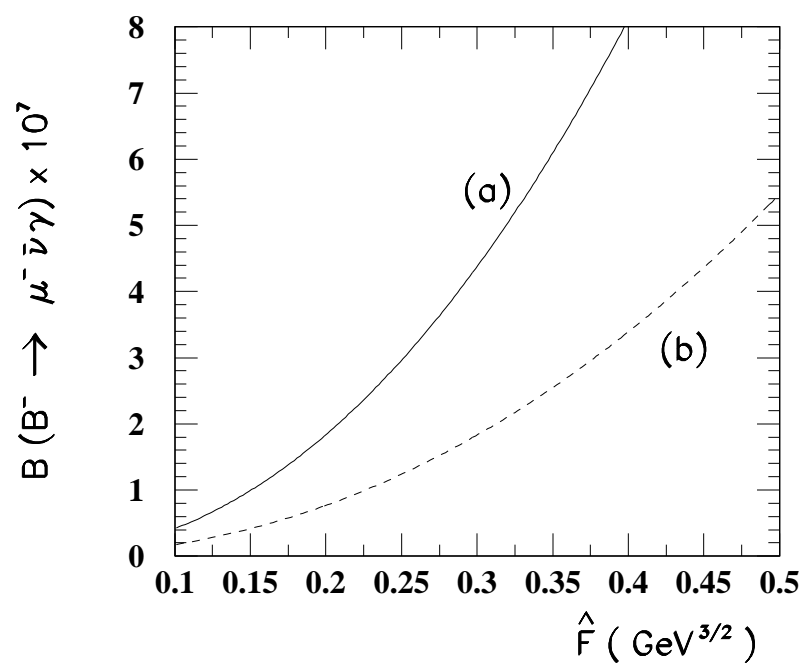


fig.2

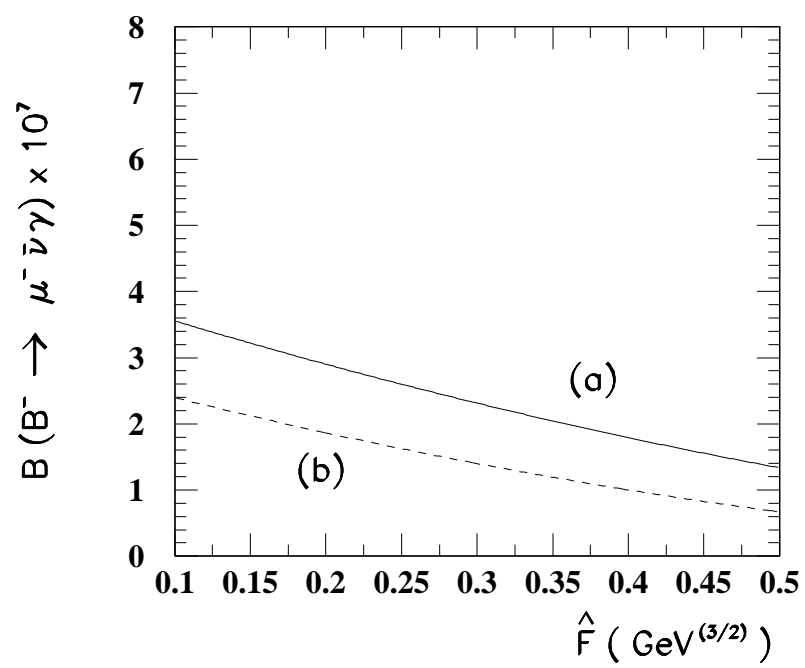


fig.3